

Fate of the true-vacuum bubbles

Jorge A. González,^{a,g} A. Bellorín,^b Mónica A. García-Ñustes,^c
L.E. Guerrero,^d S. Jiménez,^e Juan F. Marín,^{c,1} and L. Vázquez^f

^aDepartment of Physics, Florida International University,
Miami, Florida 33199, U.S.A.

^bEscuela de Física, Facultad de Ciencias, Universidad Central de Venezuela,
Apartado Postal 47586, Caracas 1041-A, Venezuela

^cInstituto de Física, Pontificia Universidad Católica de Valparaíso,
Casilla 4059, Chile

^dDepartamento de Física, Universidad Simón Bolívar,
Apartado Postal 89000, Caracas 1080-A, Venezuela

^eDepartamento de Matemática Aplicada a las T.T.II., E.T.S.I. Telecomunicación,
Universidad Politécnica de Madrid, 28040-Madrid, Spain

^fDepartamento de Matemática Aplicada, Facultad de Informática,
Universidad Complutense de Madrid, 28040-Madrid, Spain

^gDepartment of Natural Sciences, Miami Dade College,
627 SW 27th Ave., Miami, Florida 33135, U.S.A.

E-mail: jorgalbert3047@hotmail.com, alberto.bellorin@ucv.ve,
monica.garcia@pucv.cl, lguerre@usb.ve, s.jimenez@upm.es,
juanfmarin@gmail.com, lvazquez@fdi.ucm.es

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Abstract. We investigate the bounce solutions in vacuum decay problems. We show that it is possible to have a stable false vacuum in a potential that is unbounded from below.

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¹Corresponding author.

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1 Introduction

Is our vacuum metastable? Is the Universe that we live in inherently unstable?

Recent experimental and theoretical works suggest that our vacuum is probably metastable [1–23]. Measurements of Higgs boson and top quark masses indicate that there should be a “true” vacuum with less energy than the present one [1–24]. Vacuum stability problems have been discussed in many classical papers [25–36]. It is possible that the current minimum of the scalar potential is local and a deeper minimum exists or the potential has a bottomless abyss separated by a finite barrier [see figure 1]. In this situation, the Universe should eventually tunnel out into some other state, in which the elementary particles and the laws of physics are different. According to current evidence, the Universe is approximately 13.8 billion years old [37]. Scientists must find an explanation for these facts.

The framework for calculating the decay rate of a vacuum in Quantum Field Theory was developed by Coleman and collaborators [25–27], inspired by the Kobzarev et al. pioneering paper [28]. For the vacuum to decay, it must tunnel through an energy barrier, and the probability, per unit time, per unit volume for this has an exponential factor,

$$\frac{\Gamma}{V} = Ae^{-\beta/\hbar}, \quad (1.1)$$

where β is the action of a solution to the Euclidean field equations (the bounce) which interpolates between metastable (false) and true vacua. This problem is related to many phenomena in condensed matter, particle physics and cosmology [38–46].

In this article we show that it is possible to have a stable false vacuum in a potential that is unbounded from below. The remaining of the article is organized as follows. In section 2, we briefly review the Coleman’s model [25]. In section 3, we investigate the bounce solutions for some Lagrangians. In section 4, we discuss all the details of the actual evolution of the bubbles. We also present the results of numerical simulations. Finally, in section 5, we present the conclusions of our analysis.

2 The model

Consider a Quantum Field Theory described by the Lagrangian

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - U(\phi), \quad (2.1)$$

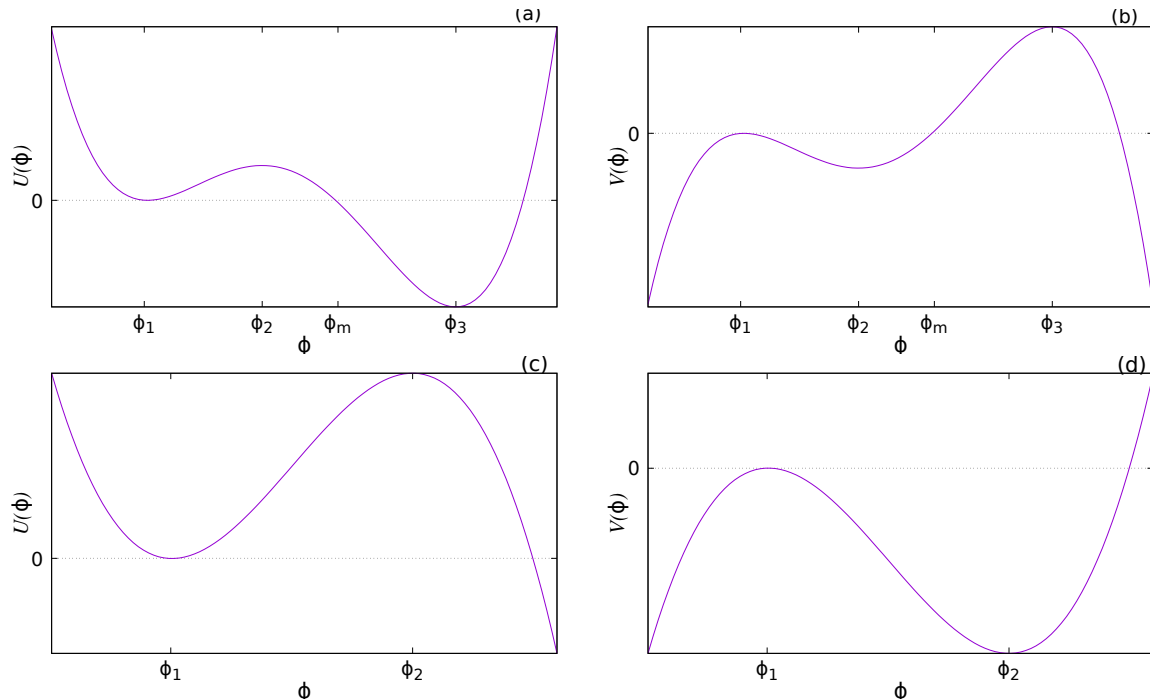


Figure 1. (a) Potential $U(\phi)$ with a “false” vacuum and a “true” vacuum. (b) Potential $V(\phi) = -U(\phi)$ for the motion of the fictitious particle. (c) Potential $U(\phi)$ showing a barrier separating a minimum (ϕ_1) and a bottomless abyss. (d) Potential $V(\phi) = -U(\phi)$, where $U(\phi)$ has the shape shown in figure 1.(c).

where $U(\phi)$ possesses two minima ϕ_1, ϕ_3 and a maximum ϕ_2 : $\phi_1 < \phi_2 < \phi_3$, $U(\phi_3) < U(\phi_1)$, $U(\phi_1) = 0$ [see figure 1.a]. The classical field equation would be

$$\phi_{tt} - \nabla^2 \phi = -\frac{dU(\phi)}{d\phi}. \quad (2.2)$$

The state $\phi = \phi_1$ is a false vacuum. It is unstable by quantum effects (e.g. barrier penetration). The accepted picture is the following. Quantum fluctuations are continually creating bubbles of true vacuum to materialize in the false vacuum. Once in a while, a bubble of true vacuum will form large enough so that it is classically energetically favorable for the bubble to grow. The bubble expands throughout the Universe converting false vacuum to true.

The Euclidean equation of motion is

$$\phi_{\tau\tau} + \nabla^2 \phi = \frac{dU}{d\phi}, \quad (2.3)$$

where $\tau = it$ is the imaginary time. The boundary conditions of the bounce solution are

$$\phi_\tau(0, \mathbf{x}) = 0, \quad (2.4)$$

$$\lim_{\tau \rightarrow \pm\infty} \phi(\tau, \mathbf{x}) = \phi_1. \quad (2.5)$$

The integral that yields the action of the bounce is

$$\beta = \int d\tau d^3x \left[\frac{1}{2} \phi_\tau^2 + \frac{1}{2} (\nabla \phi)^2 + U \right]. \quad (2.6)$$

The action β will be finite only if

$$\lim_{|\mathbf{x}| \rightarrow \infty} \phi(\tau, x) = \phi_1. \quad (2.7)$$

The bounce solution is invariant under four-dimensional Euclidean rotations. Define

$$\rho \equiv (|\mathbf{x}|^2 + \tau^2)^{1/2}. \quad (2.8)$$

The bounce solution is a function of ρ only. From equation (2.3), we obtain

$$\frac{d^2 \phi}{d\rho^2} + \frac{3}{\rho} \frac{d\phi}{d\rho} = -\frac{dV(\phi)}{d\phi}, \quad (2.9)$$

where $V(\phi) = -U(\phi)$ [see figure 1.b]. The conditions for the bounce solutions are now

$$\phi_\rho(0) = 0, \quad (2.10)$$

$$\lim_{\rho \rightarrow \infty} \phi(\rho) = \phi_1. \quad (2.11)$$

The action β can be calculated using the formula

$$\beta = 2\pi^2 \int_0^\infty d\rho \rho^3 \left[\frac{1}{2} \phi_\rho^2 + U(\phi) \right]. \quad (2.12)$$

We can consider equation (2.9) as the Newton's equation of motion for a fictitious particle of unit mass moving in the potential $V(\phi)$ [see figure 1.b], where ϕ is the “particle” position and ρ is the “time”. There is a dissipative force acting on the particle

$$F_{\text{dis}} = -\frac{3}{\rho} \frac{d\phi}{d\rho}. \quad (2.13)$$

The “particle” must be released at rest at time zero. The initial condition must be

$$\phi_m < \phi(0) < \phi_3. \quad (2.14)$$

For the bounce solution to exist, the “particle” should come to rest at “time” infinity at ϕ_1 (i.e. on top of the maximum $V(\phi_1)$) [see figure 1.b].

In many articles, the authors use the so-called thin-wall approximation [25] and the ϕ^4 theory as a model. The employed solution is nothing else but the one-dimensional ϕ^4 soliton. This approach loses all the complexities of the 3+1 dimensional nonlinear field equations. In the present paper we will show that, for some Lagrangians, the action of the bounce solution can be arbitrarily large.

3 The bounce solution

Let us return to the motion of the fictitious particle in the potential $V(\phi)$ [see figure 1.b]. A particle that starts its motion at a point $\phi(0) > \phi_m$ should pass the point $\phi = \phi_2$ (at the bottom of the valley) in order to reach the point $\phi = \phi_1$. We can expect that in a vicinity of point ϕ_2 the particle makes oscillations. This is true when the potential $V(\phi)$ behaves as $V(\phi) \sim (\phi_2 - \phi)^2/2$ in a neighborhood of point $\phi = \phi_2$, and we are studying the

one-dimensional case. For the $D = 3 + 1$ case, the particle will make damped oscillations near the point $\phi = \phi_2$. Nevertheless, the bounce solution can still exist.

Suppose there are potentials for which the motion of the “particle” in a vicinity of $\phi = \phi_2$ will be in the overdamped regime. Then if the particle starts its motion at a point $\phi(0) > \phi_2$, there will be an overdamped trajectory that will end at point $\phi = \phi_2$ for $\rho \rightarrow \infty$. The particle will never visit the points for which $\phi < \phi_2$.

Let us define

$$U(\phi) = \begin{cases} \frac{a_1(\phi - \phi_1)^2}{2}, & \text{for } \phi < \phi_{12}, \\ -\frac{(\phi - \phi_2)^4}{4} + \Delta_2, & \text{for } \phi_{12} < \phi, \end{cases} \quad (3.1)$$

where $\phi = \phi_{12}$ is a point for which $U(\phi)$ and its first derivative are continuous, i.e. $a_1(\phi_{12} - \phi_1)^2/2 = \Delta_2 - (\phi_{12} - \phi_2)^4/4$, and $a_1(\phi_{12} - \phi_1) = -(\phi_{12} - \phi_2)^3$. All the solutions with the conditions

$$\phi(0) > \phi_2, \quad (3.2a)$$

$$\left. \frac{d\phi}{d\rho} \right|_{\rho=0} = 0, \quad (3.2b)$$

are given by the family of functions

$$\phi(\rho) = \phi_2 + \frac{Q}{1 + \frac{Q^2}{8}\rho^2}, \quad (3.3)$$

where $\phi(0) = \phi_2 + Q$. This includes solutions with the initial conditions

$$\phi(0) > \phi_m, \quad (3.4a)$$

$$\left. \frac{d\phi}{d\rho} \right|_{\rho} = 0. \quad (3.4b)$$

Note that the solution (3.3) is the most general solution satisfying the conditions (3.4a) and (3.4b). If there is another solution, there is no “unicity”. Note also that there exists a continuum of solutions provided by (3.3), specified by any initial condition $\phi(0) > \phi_2$.

It does not matter how high the “particle” will be at $\rho = 0$ in the potential $V(\phi)$ [see figure 1.b], the particle will be stuck at $\phi = \phi_2$ even for $\rho \rightarrow \infty$. It will never visit points close to $\phi = \phi_1$. The action of all these solutions is infinite.

In general, for any potential that behaves as $U(\phi) \simeq (\phi - \phi_2)^4/4$ in a neighborhood of $\phi = \phi_2$, the motion of the “particle” for $D = 3 + 1$ will be difficult near the point $\phi = \phi_2$.

Consider a generalization to the equation (2.9) in a D -dimensional space-time

$$\frac{d^2\phi}{d\rho^2} + \frac{D-1}{\rho} \frac{d\phi}{d\rho} = -\frac{dV(\phi)}{d\phi}, \quad (3.5)$$

where

$$U(\phi) = -V(\phi) = \frac{a|\phi|^3}{3} - \frac{b|\phi|^4}{4}, \quad a > 0, b > 0. \quad (3.6)$$

For this potential, $\phi_1 = 0$, and $\phi_2 = a/b$. We can find the exact bounce solution to equation (3.5)

$$\phi_{\text{bounce}} = \frac{Q}{1 + N\rho^2}, \quad (3.7)$$

where

$$Q = \frac{4}{4-D} \frac{a}{b}, \quad (3.8a)$$

$$N = \frac{2}{(4-D)^2} \frac{a^2}{b}. \quad (3.8b)$$

Notice that the bounce solution is unique in the sense that there is only one initial condition $\phi(0) = 4a/(4-D)b$ for which $\lim_{\rho \rightarrow \infty} \phi(\rho) = \phi_1 \equiv 0$.

We would like to remark that as D is increased from $D = 1$, the ‘‘amplitude’’ Q of the bounce will increase (see eq. (3.8a)) ($\phi(0) = Q$). In other words, we need an initial condition with enough potential energy for the ‘‘particle’’ to be able to climb the hill and reach the point $\phi = \phi_1$. For larger D , there is more dissipation. So the particle needs more energy. However, $Q \rightarrow \infty$ as $D \rightarrow 4$ (!). Something very important happens when $D = 3 + 1$. This is an indication that for $D = 4$, it is very difficult for the ‘‘particle’’ to arrive at point $\phi = \phi_1 = 0$. This can be also an evidence that

$$\lim_{D \rightarrow 4} \beta_{(\text{bounce})} = \infty. \quad (3.9)$$

This implies that there exists no finite bounce solution for $D = 4$.

It is interesting to study Lagrangians where the potential $U(\phi)$ behaves as

$$U(\phi) \simeq \frac{(\phi - \phi_1)^{n_1}}{n_1}, \quad \text{near } \phi = \phi_1, \quad (3.10a)$$

$$U(\phi) \simeq \frac{(\phi_2 - \phi)^{n_2}}{n_2} + \Delta_2, \quad \text{near } \phi = \phi_2, \quad (3.10b)$$

and

$$U(\phi) \simeq \frac{(\phi - \phi_3)^{n_3}}{n_3} - \Delta_3, \quad \text{near } \phi = \phi_3, \quad (3.10c)$$

where $n_1 \geq 2$, $n_2 \geq 2$, and $n_3 \geq 2$. As n_1 , n_2 , n_3 and the dimension D increase, the probability of the formation of bubbles of the ‘‘true’’ vacuum inside the ‘‘false’’ vacuum that will expand approaches zero. Compare the results of this section with the results discussed in ref. [47].

In brief, we gave for the first time, a mathematical proof that it is possible to have a stable false vacuum in a potential that is unbounded from below.

4 Discussion

Consider a chain of coupled pendula. In the continuum limit, this system can be described by the sine-Gordon equation

$$\phi_{tt} - \phi_{xx} + \sin \phi = 0. \quad (4.1)$$

The static kink solution to equation (4.1) is the following

$$\phi_k = 4 \arctan[\exp(x)]. \quad (4.2)$$

This solution is stable [48–52]. However, note that there are pendula that are completely outside the stable equilibrium $\phi = 0$ ($\phi = 2\pi$). In fact, for the point $x = 0$, the pendulum is at position $\phi = \pi$, which, for a single pendulum, would be a completely unstable position.

In the solution given by equation (4.2), the pendulum at position $\phi = \pi$ and those that are close to that one are sustained by the majority of the pendula, which are close to the stable position $\phi = 0$ ($\phi = 2\pi$).

Consider now a chain of linked small balls, which are moving in the potential

$$U(\phi) = \frac{\phi^2}{2} - \frac{\phi^3}{3}. \quad (4.3)$$

The graph of this function is similar to figure 1.c. The equation of motion is

$$\phi_{tt} - \phi_{xx} = -\phi + \phi^2, \quad (4.4)$$

whose exact critical-bubble solution [51] can be obtained as

$$\phi(x) = \frac{3}{2 \cosh^2(x/2)}. \quad (4.5)$$

The unstable equilibrium of the potential is $\phi_2 = 1$. For instance, an initial condition of the form

$$\phi(x, 0) = \frac{1.4}{\cosh^2(x)}, \quad (4.6a)$$

$$\phi_t(x, 0) = 0, \quad (4.6b)$$

would lead to a dynamics where the balls fall to the “left” where the potential well ($\phi_1 = 0$) exists. There are balls situated to the “right” of the unstable equilibrium. However, these balls will not fall to the “right” (to the “abyss”). The ball outside the potential well are sustained by the balls that are inside the potential well. We should say that “larger” bubbles like the following

$$\phi(x, 0) = \frac{1.6}{\cosh^2(x/4)}, \quad (4.7)$$

will grow for ever.

Let us examine the following equation

$$\phi_{tt} - \nabla^2 \phi = F(\phi), \quad (4.8)$$

where $F(\phi) = a[\phi(\phi+2)(2+\delta-\phi)]^{2n-1}$, and $n \geq 3$. The potential $U(\phi)$ ($F(\phi) = -dU(\phi)/d\phi$) for equation (4.8) has the shape shown in figure 1.a with two minima (a “false” vacuum and a “true” vacuum) and the properties discussed in section 3. In $D = 3 + 1$ dimensions, the “balls” that are close to the unstable equilibrium position $\phi_2 = 0$ are capable to sustain the “balls” that are very far away from the equilibrium (including amplitudes that are arbitrarily large). Initial conditions like the following

$$\phi(x, y, z, 0) = \phi_o + \frac{Q}{[1 + b(x^2 + y^2 + z^2)]^{1/2}}, \quad (4.9a)$$

$$\phi_t(x, y, z, 0) = 0, \quad (4.9b)$$

with $Q > 0$ and $b > 0$, where all the “balls” are outside the unstable equilibrium, do not lead to an evolution with the “balls” falling to the “right”.

Note that there are “bubbles”, whose action is infinite and are made mostly of “true” vacuum, but, nevertheless, do not grow. This means they do not expand throughout the

Universe converting false vacuum to true. This leads to the conclusion that “normal” bubbles with finite action and made of “true” vacuum inside the Universe of “false” vacuum will not grow either.

Let us study an initial condition where the majority of the “balls” are close to the locally stable equilibrium $\phi = \phi_1$. In $D = 3 + 1$, the system is able to sustain any field configuration with the values of ϕ as far to the right as we wish from the unstable equilibrium $\phi = \phi_2$. In $D = 3 + 1$ the link between the “balls” is so strong that the “balls” outside the equilibrium cannot drag those that are inside the equilibrium. Initial conditions in the form

$$\phi(x, y, z, 0) = -2 + A [\tanh[B(r + d)] - \tanh[B(r - d)]], \quad (4.10a)$$

$$\phi_t(x, y, z, 0) = 0, \quad (4.10b)$$

will not evolve in such a way that the bubble will expand, and the false vacuum will be converted to the “true” vacuum.

Figure 2 shows that most bubbles made of “true” vacuum inside a Universe made of “false” vacuum will collapse very fast. There is a relaxation process such that $\lim_{t \rightarrow \infty} \phi = \phi_1$.

Figure 3.a shows the dynamics of a “bubble” made completely of “true vacuum”. This bubble is stationary and marginally stable. There is a whole continuum “zone” in configuration space with bubbles of different amplitudes and different behaviors of the tails. All the bubbles in this “zone” are stationary solutions to the field equations. These bubbles are not asymptotically stable. A perturbation of the bubble will lead to a different stationary configuration that belongs to the mentioned “zone”. However, the most remarkable fact is that these bubbles do not expand!

Figure 3.b shows the dynamics of a very large bubble made of “true” vacuum inside a Universe of “false” vacuum. This bubble does not expand. However, it is so large that its collapse is very slow. This bubble can be considered a quasi-stationary long-lived structure that could contain “new physics”. The properties and physical meaning of these structures will be discussed elsewhere. All bubbles of the “true” vacuum inside a Universe of the “false” vacuum will collapse.

As a final remark, note that solutions (3.3) are not bounce. We have discussed them for some given potentials in section 3. However, we have a system with an exact bounce solution (given by equations (3.7) and (3.8)) which is a unique solution. There is only one initial condition such that for ρ approaching infinity, ϕ will be approaching ϕ_1 . The bounce solution exists for $D < 4$. However, as $D \rightarrow 4$, $Q \rightarrow \infty$. This shows that $\lim_{D \rightarrow 4} \beta_{bounce} = \infty$. Indeed, numerical investigations with the ordinary differential equation (2.9) using the Strauss-Vazquez method [53, 54] show that the “particle” never reaches point $\phi = \phi_1$. Additionally, using the general partial differential equation (4.8) we have shown that the bubbles of true vacuum never expand. The potentials involved in these problems have degenerate critical points in the sense of René Thom [55], and the solutions near such points have special properties. This kind of potentials are relevant when we are near criticality [6, 8].

5 Conclusions

We have investigated the bounce solutions in some classes of vacuum decay problems. We have obtained a rigorous mathematical proof of the fact that it is possible to have a stable false vacuum in a potential that is unbounded from below (See section 3). This is our actual and main result. Additionally, using the analysis of the results discussed in sections 3 and 4,

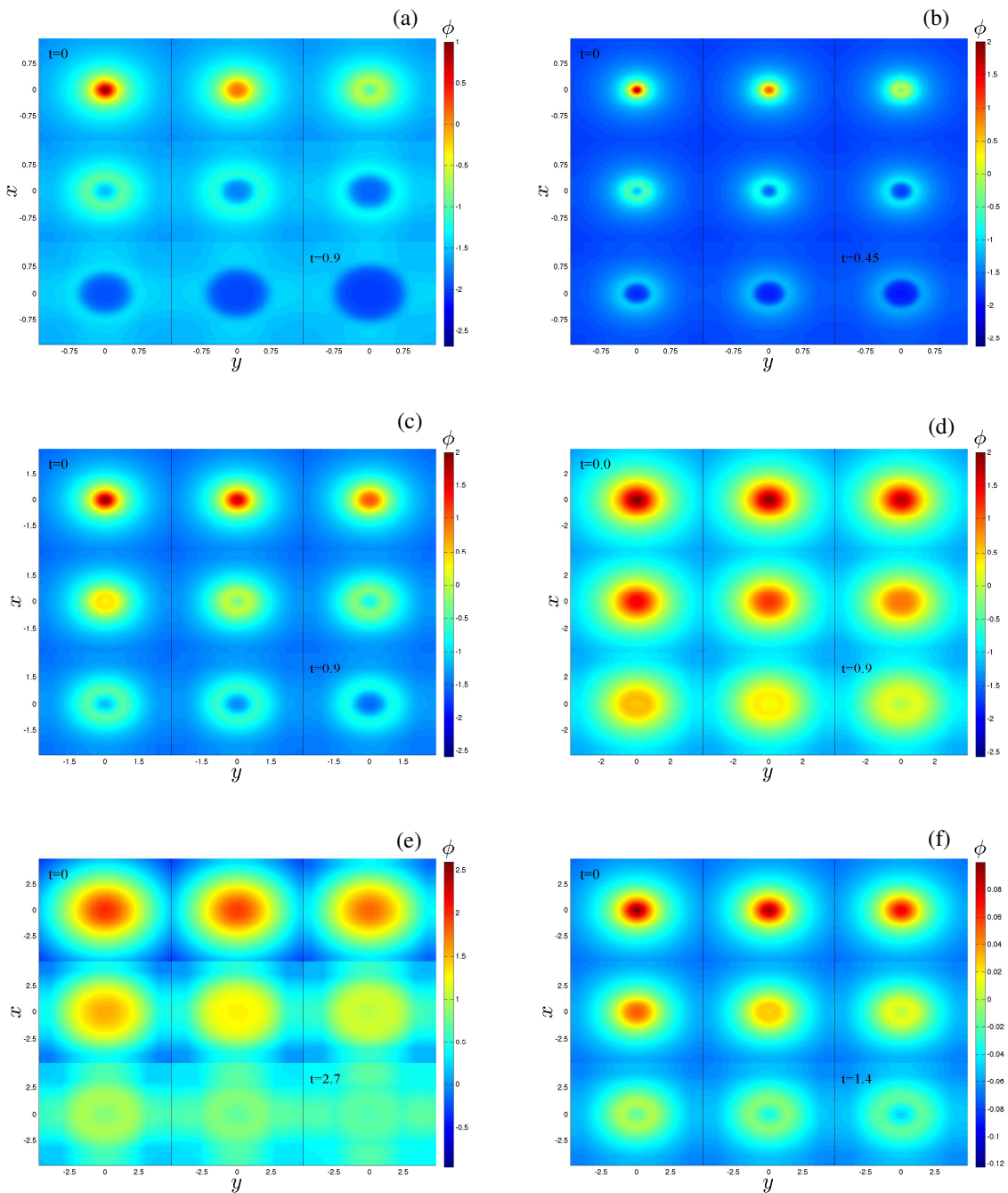


Figure 2. Collapse of bubbles with different contents of “true” vacuum inside a Universe made of “false” vacuum. Results obtained from the numerical solution of eq. (4.8) in 3 dimensions with the initial conditions of eq. (4.9) for (a) $\phi_o = -2$, $Q = 3$, and $b = 27$. (b) $\phi_o = -2$, $Q = 4$, and $b = 85$. (c) $\phi_o = -2$, $Q = 4.001$, and $b = 5$. (d) $\phi_o = -2$, $Q = 4.005$, and $b = 1$. (e) $\phi_o = -2$, $Q = 4.001$, and $b = 0.1$. (f) $\phi_o = -0.1$, $Q = 0.2$, and $b = 1$. Only the time evolution of the field at $z = 0$ is showed. Any other profile will exhibit a similar behavior due to the symmetry of the system.

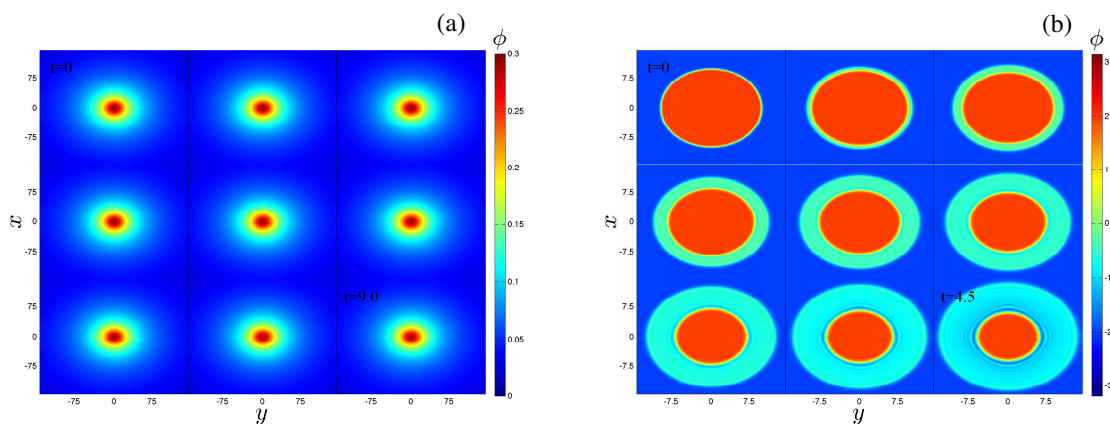


Figure 3. (a) A stationary bubble in 3 dimensions mostly made of true vacuum. Results from numerical simulations of eq. (4.8) for the initial conditions of eq. (4.9) for $\phi_o = 0$, $Q = 0.3$, and $b = 0.0027$. (b) Results from numerical simulations in 3 dimensions for the initial condition of eq. (4.10) for $B = 5$, $d = 10$ and $A = 2.0005$. Initially there is a partial collapse of the walls of the field configuration, but there is no bubble of true vacuum that expands throughout the Universe. Only the time evolution of the field at $z = 0$ is shown. Any other profile will exhibit a similar behavior due to the symmetry of the system.

and the numerical simulations, we can formulate several conjectures. Note that the potential used in the numerical simulations (eq. (4.8)) has the shape shown in figure 1.a with two finite minima (a “false” vacuum and a “true” vacuum).

- We conjecture that when the critical points of potential $U(\phi)$ are near criticality [6, 8] (see sections 3 and 4), the bounce solution does not exist or the action of the bounce solution is arbitrarily large. So the probability of the “false” vacuum decay is exponentially small.
- We conjecture that under the above conditions the bubbles of “true” vacuum inside a Universe of “false” vacuum will not grow. Thus there is no bubble expansion that would convert the “false” vacuum to “true”.
- We conjecture (after Turner and Wilczek [32]) that “It is possible that our present vacuum is metastable, and that, nevertheless, the Universe would have chosen to get ‘hung up’ in it”.

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References

- [1] B. Grinstein and C.W. Murphy, *Semiclassical Approach to Heterogeneous Vacuum Decay*, *JHEP* **12** (2015) 063 [[arXiv:1509.05405](#)] [[INSPIRE](#)].
- [2] A. Sen, *Riding Gravity Away from Doomsday*, *Int. J. Mod. Phys. D* **24** (2015) 1544004 [[arXiv:1503.08130](#)] [[INSPIRE](#)].
- [3] P. Burda, R. Gregory and I. Moss, *Gravity and the stability of the Higgs vacuum*, *Phys. Rev. Lett.* **115** (2015) 071303 [[arXiv:1501.04937](#)] [[INSPIRE](#)].
- [4] Y. Ema, K. Mukaida and K. Nakayama, *Fate of Electroweak Vacuum during Preheating*, *JCAP* **10** (2016) 043 [[arXiv:1602.00483](#)] [[INSPIRE](#)].
- [5] V. Hoang, P.Q. Hung and A.S. Kamat, *Non-sterile electroweak-scale right-handed neutrinos and the dual nature of the 125-GeV scalar*, *Nucl. Phys. B* **896** (2015) 611 [[arXiv:1412.0343](#)] [[INSPIRE](#)].
- [6] G. Degrassi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori et al., *Higgs mass and vacuum stability in the Standard Model at NNLO*, *JHEP* **08** (2012) 098 [[arXiv:1205.6497](#)] [[INSPIRE](#)].
- [7] P. Burda, R. Gregory and I. Moss, *The fate of the Higgs vacuum*, *JHEP* **06** (2016) 025 [[arXiv:1601.02152](#)] [[INSPIRE](#)].
- [8] D. Buttazzo, G. Degrassi, P.P. Giardino, G.F. Giudice, F. Sala, A. Salvio et al., *Investigating the near-criticality of the Higgs boson*, *JHEP* **12** (2013) 089 [[arXiv:1307.3536](#)] [[INSPIRE](#)].
- [9] J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Riotto and A. Strumia, *Higgs mass implications on the stability of the electroweak vacuum*, *Phys. Lett. B* **709** (2012) 222 [[arXiv:1112.3022](#)] [[INSPIRE](#)].
- [10] ATLAS collaboration, *Combined search for the Standard Model Higgs boson using up to 4.9 fb⁻¹ of pp collision data at $\sqrt{s} = 7$ TeV with the ATLAS detector at the LHC*, *Phys. Lett. B* **710** (2012) 49 [[arXiv:1202.1408](#)] [[INSPIRE](#)].
- [11] CMS collaboration, *Combined results of searches for the standard model Higgs boson in pp collisions at $\sqrt{s} = 7$ TeV*, *Phys. Lett. B* **710** (2012) 26 [[arXiv:1202.1488](#)] [[INSPIRE](#)].
- [12] A. Andreassen, W. Frost and M.D. Schwartz, *Consistent Use of the Standard Model Effective Potential*, *Phys. Rev. Lett.* **113** (2014) 241801 [[arXiv:1408.0292](#)] [[INSPIRE](#)].
- [13] ATLAS collaboration, *Measurement of the top quark mass in the $t\bar{t} \rightarrow \text{lepton} + \text{jets}$ and $t\bar{t} \rightarrow \text{dilepton}$ channels using $\sqrt{s} = 7$ TeV ATLAS data*, *Eur. Phys. J. C* **75** (2015) 330 [[arXiv:1503.05427](#)] [[INSPIRE](#)].
- [14] CMS collaboration, *Measurement of the top quark mass using proton-proton data at $\sqrt{s} = 7$ and 8 TeV*, *Phys. Rev. D* **93** (2016) 072004 [[arXiv:1509.04044](#)] [[INSPIRE](#)].
- [15] F. Bezrukov, M. Yu. Kalmykov, B.A. Kniehl and M. Shaposhnikov, *Higgs Boson Mass and New Physics*, *JHEP* **10** (2012) 140 [[arXiv:1205.2893](#)] [[INSPIRE](#)].
- [16] ATLAS collaboration, *Measurement of the top quark mass in the $t\bar{t} \rightarrow \text{dilepton}$ channel from $\sqrt{s} = 8$ TeV ATLAS data*, *Phys. Lett. B* **761** (2016) 350 [[arXiv:1606.02179](#)] [[INSPIRE](#)].
- [17] A. Shkerin and S. Sibiryakov, *On stability of electroweak vacuum during inflation*, *Phys. Lett. B* **746** (2015) 257 [[arXiv:1503.02586](#)] [[INSPIRE](#)].
- [18] V. Branchina, E. Messina and M. Sher, *Lifetime of the electroweak vacuum and sensitivity to Planck scale physics*, *Phys. Rev. D* **91** (2015) 013003 [[arXiv:1408.5302](#)] [[INSPIRE](#)].
- [19] N. Khan and S. Rakshit, *Study of electroweak vacuum metastability with a singlet scalar dark matter*, *Phys. Rev. D* **90** (2014) 113008 [[arXiv:1407.6015](#)] [[INSPIRE](#)].

- [20] S.-F. Ge, H.-J. He, J. Ren and Z.-Z. Xianyu, *Realizing Dark Matter and Higgs Inflation in Light of LHC Diphoton Excess*, *Phys. Lett. B* **757** (2016) 480 [[arXiv:1602.01801](#)] [[INSPIRE](#)].
- [21] A. Kusenko, L. Pearce and L. Yang, *Postinflationary Higgs relaxation and the origin of matter-antimatter asymmetry*, *Phys. Rev. Lett.* **114** (2015) 061302 [[arXiv:1410.0722](#)] [[INSPIRE](#)].
- [22] A. Hook, J. Kearney, B. Shakya and K.M. Zurek, *Probable or Improbable Universe? Correlating Electroweak Vacuum Instability with the Scale of Inflation*, *JHEP* **01** (2015) 061 [[arXiv:1404.5953](#)] [[INSPIRE](#)].
- [23] A.V. Bednyakov, B.A. Kniehl, A.F. Pikelner and O.L. Veretin, *Stability of the Electroweak Vacuum: Gauge Independence and Advanced Precision*, *Phys. Rev. Lett.* **115** (2015) 201802 [[arXiv:1507.08833](#)] [[INSPIRE](#)].
- [24] A. Kusenko, *Are we on the brink of the Higgs abyss?*, *Physics* **8** (2015) 108.
- [25] S.R. Coleman, *The Fate of the False Vacuum. 1. Semiclassical Theory*, *Phys. Rev. D* **15** (1977) 2929 [*Erratum ibid.* **D 16** (1977) 1248] [[INSPIRE](#)].
- [26] C.G. Callan Jr. and S.R. Coleman, *The Fate of the False Vacuum. 2. First Quantum Corrections*, *Phys. Rev. D* **16** (1977) 1762 [[INSPIRE](#)].
- [27] S.R. Coleman and F. De Luccia, *Gravitational Effects on and of Vacuum Decay*, *Phys. Rev. D* **21** (1980) 3305 [[INSPIRE](#)].
- [28] I. Yu. Kobzarev, L.B. Okun and M.B. Voloshin, *Bubbles in Metastable Vacuum*, *Sov. J. Nucl. Phys.* **20** (1975) 644 [[INSPIRE](#)].
- [29] A. Gorsky, A. Mironov, A. Morozov and T.N. Tomaras, *Is the Standard Model saved asymptotically by conformal symmetry?*, *J. Exp. Theor. Phys.* **120** (2015) 344 [[arXiv:1409.0492](#)] [[INSPIRE](#)].
- [30] K. Blum, R.T. D’Agnolo and J. Fan, *Vacuum stability bounds on Higgs coupling deviations in the absence of new bosons*, *JHEP* **03** (2015) 166 [[arXiv:1502.01045](#)] [[INSPIRE](#)].
- [31] G. Isidori, G. Ridolfi and A. Strumia, *On the metastability of the standard model vacuum*, *Nucl. Phys. B* **609** (2001) 387 [[hep-ph/0104016](#)] [[INSPIRE](#)].
- [32] M.S. Turner and F. Wilczek, *Might our vacuum be metastable?*, *Nature* **298** (1982) 633 [[INSPIRE](#)].
- [33] M. Lindner, M. Sher and H.W. Zaglauer, *Probing Vacuum Stability Bounds at the Fermilab Collider*, *Phys. Lett. B* **228** (1989) 139 [[INSPIRE](#)].
- [34] M. Sher, *Electroweak Higgs Potentials and Vacuum Stability*, *Phys. Rept.* **179** (1989) 273 [[INSPIRE](#)].
- [35] I.V. Krive and A.D. Linde, *On the Vacuum Stability Problem in Gauge Theories*, *Nucl. Phys. B* **117** (1976) 265 [[INSPIRE](#)].
- [36] N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, *Bounds on the Fermions and Higgs Boson Masses in Grand Unified Theories*, *Nucl. Phys. B* **158** (1979) 295 [[INSPIRE](#)].
- [37] PLANCK collaboration, P.A.R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13 [[arXiv:1502.01589](#)] [[INSPIRE](#)].
- [38] A.G. Cohen, D.B. Kaplan and A.E. Nelson, *Progress in electroweak baryogenesis*, *Ann. Rev. Nucl. Part. Sci.* **43** (1993) 27 [[hep-ph/9302210](#)] [[INSPIRE](#)].
- [39] M. Trodden, *Electroweak baryogenesis*, *Rev. Mod. Phys.* **71** (1999) 1463 [[hep-ph/9803479](#)] [[INSPIRE](#)].
- [40] K. Yagi, T. Hatsuda and Y. Miake, *Quark-Gluon Plasma*, Cambridge University Press, Cambridge, U.K. (2005).

- [41] A. Linde, *Particle Physics and Inflationary Cosmology*, CRC Press (1990).
- [42] S. Weinberg, *Cosmology*, Oxford University Press, Oxford, U.K. (2008).
- [43] V.A. Gani, M.A. Lizunova and R.V. Radomskiy, *Scalar triplet on a domain wall: an exact solution*, *JHEP* **04** (2016) 043 [[arXiv:1601.07954](#)] [[INSPIRE](#)].
- [44] P. Hänggi, F. Marchesoni and P. Sodano, *Nucleation of thermal sine-Gordon solitons: Effect of many-body interactions*, *Phys. Rev. Lett.* **60** (1988) 2563.
- [45] J. Aubry, *A unified approach to the interpretation of displacive and order-disorder systems. I. Thermodynamical aspect*, *J. Chem. Phys.* **62** (1975) 3217.
- [46] F. Marchesoni, C. Cattuto and G. Costantini, *Elastic strings in solids: Thermal nucleation*, *Phys. Rev.* **B 57** (1998) 7930.
- [47] A. Aravind, B.S. DiNunno, D. Lorschbough and S. Paban, *Analyzing multifield tunneling with exact bounce solutions*, *Phys. Rev.* **D 91** (2015) 025026 [[arXiv:1412.3160](#)] [[INSPIRE](#)].
- [48] J.A. González, B.A. Mello, L.I. Reyes and L.E. Guerrero, *Resonance phenomena of a solitonlike extended object in a bistable potential*, *Phys. Rev. Lett.* **80** (1998) 1361.
- [49] J.A. González, A. Bellorín and L.E. Guerrero, *Internal modes of sine-Gordon solitons in the presence of spatiotemporal perturbations*, *Phys. Rev.* **E 65** (2002) 065601.
- [50] J.A. González, S. Cuenda and A. Sánchez, *Kink dynamics in spatially inhomogeneous media: The role of internal modes*, *Phys. Rev.* **E 75** (2007) 036611.
- [51] F. Oliveira and J.A. González, *Bond-stability criterion in chain dynamics*, *Phys. Rev.* **B 54** (1996) 3954.
- [52] M.A. García-Ñustes and J.A. González, *Formation of a two-kink soliton pair in perturbed sine-Gordon models due to kinkinternal-mode instabilities*, *Phys. Rev.* **E 86** (2012) 066602 [[INSPIRE](#)].
- [53] W. Strauss and L. Vazquez, *Numerical Solution of a Nonlinear Klein-Gordon Equation*, *J. Comput. Phys.* **28** (1978) 271 [[INSPIRE](#)].
- [54] S. Jiménez, J.A. González and L. Vázquez, *Fractional Duffing's Equation and Geometrical Resonance*, *Int. J. Bifurcat. Chaos* **23** (2013) 1350089.
- [55] R. Thom, *Structural Stability and Morphogenesis*, W.A. Benjamin, Reading, Massachusetts (1975).